

Exercise 10.1 (Revised) - Chapter 10 - Circles - Ncert Solutions class 10 - Maths

Updated On 11-02-2025 By Lithanya

Chapter 10: Circles - NCERT Solutions for Class 10 Maths

Ex 10.1 Question 1.

How many tangents can a circle have?

Answer.

A circle can have infinitely many tangents since there are infinitely many points on the circumference of the circle and at each point of it, it has a unique tangent.

Ex 10.1 Question 2.

Fill in the blanks:

- (i) A tangent to a circle intersects it in _____ point(s).
- (ii) A line intersecting a circle in two points is called a _____
- (iii) A circle can have _____ parallel tangents at the most.
- (iv) The common point of a tangent to a circle and the circle is called _____

Answer.

- (i) A tangent to a circle intersects it in exactly one point.
- (ii) A line intersecting a circle in two points is called a secant.
- (iii) A circle can have two parallel tangents at the most.
- (iv) The common point of a tangent to a circle and the circle is called point of contact.

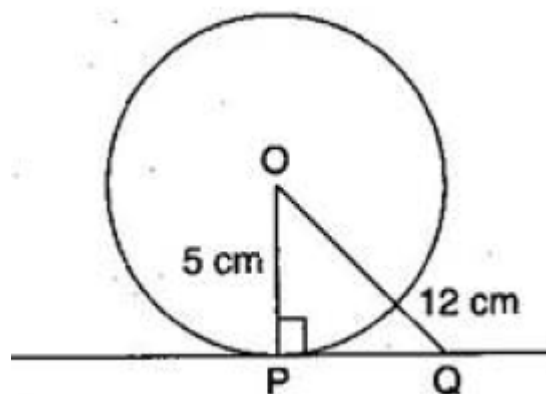
Ex 10.1 Question 3.

A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that $OQ = 12$ cm. Length PQ is:

- (A) 12 cm
- (B) 13 cm
- (C) 8.5 cm
- (D) $\sqrt{119}$ cm

Answer.

(D) $\because PQ$ is the tangent and OP is the radius through the point of contact.
 $\therefore \angle OPQ = 90^\circ$ [The tangent at any point of a circle is $\perp$ to the radius through the point of contact]
 $\therefore$ In right triangle OPQ ,



$$OQ^2 = OP^2 + PQ^2 \text{ [By Pythagoras theorem]}$$

$$\Rightarrow (12)^2 = (5)^2 + PQ^2$$

$$\Rightarrow 144 = 25 + PQ^2$$

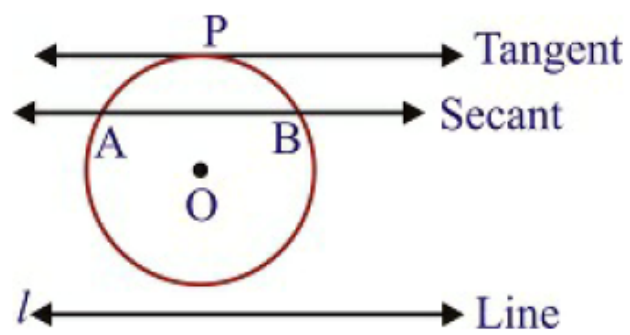
$$\Rightarrow PQ^2 = 144 - 25 = 119$$

$$\Rightarrow PQ = \sqrt{119} \text{ cm}$$

Ex 10.1 Question 4.

Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Answer:



Exercise 10.2 (Revised) - Chapter 10 - Circles - Ncert Solutions class 10 - Maths

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Chapter 10: Circles - NCERT Solutions for Class 10 Maths

Ex 10.2 Question 1.

From a point Q , the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is:

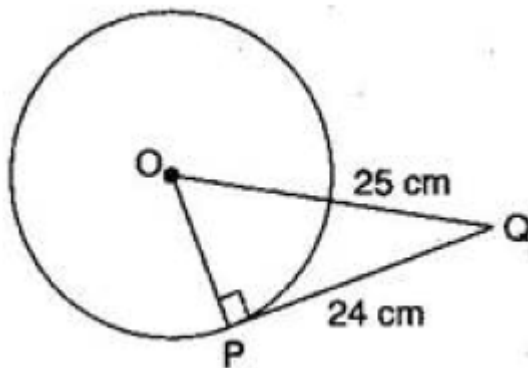
- (A) 7 cm
- (B) 12 cm
- (C) 15 cm
- (D) 24.5 cm

Answer.

(A)

$\therefore \angle OPQ = 90^\circ$

[The tangent at any point of a circle is perpendicular to the radius through the point of contact]



$\therefore$ In right triangle OPQ,

$$OQ^2 = OP^2 + PQ^2$$

[By Pythagoras theorem]

$$\Rightarrow (25)^2 = OP^2 + (24)^2$$

$$\Rightarrow 625 = OP^2 + 576$$

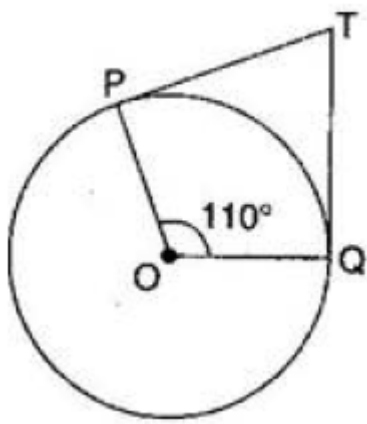
$$\Rightarrow OP^2 = 625 - 576 = 49$$

$$\Rightarrow OP = 7 \text{ cm}$$

Ex 10.2 Question 2.

In figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110^\circ$, then $\angle PTQ$ is equal to:





- (A) 60°
 (B) 70°
 (C) 80°
 (D) 90°

Answer.

(B)

$$\angle POQ = 110^\circ, \angle OPT = 90^\circ \text{ and } \angle OQT = 90^\circ$$

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

In quadrilateral OPTQ

$$\angle POQ + \angle OPT + \angle OQT + \angle PTQ = 360^\circ$$

[Angle sum property of quadrilateral]

$$\Rightarrow 110^\circ + 90^\circ + 90^\circ + \angle PTQ = 360^\circ$$

$$\Rightarrow 290^\circ + \angle PTQ = 360^\circ$$

$$\Rightarrow \angle PTQ = 360^\circ - 290^\circ$$

$$\Rightarrow \angle PTQ = 70^\circ$$

Ex 10.2 Question 3.

If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80° , then $\angle POA$ is equal to:

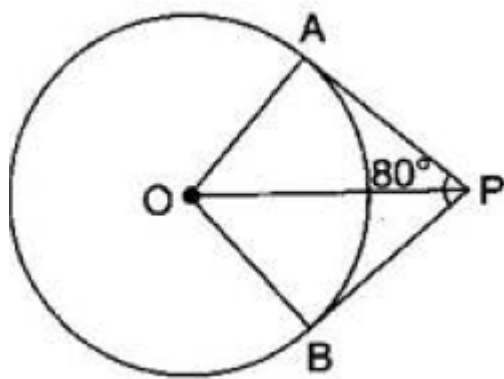
- (A) 50°
 (B) 60°
 (C) 70°
 (D) 80°

Answer.

(A)

$$\therefore \angle OAP = 90^\circ$$

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]



$$\angle OPA = \frac{1}{2} \angle BPA = \frac{1}{2} \times 80^\circ = 40^\circ$$

[Centre lies on the bisector of the angle between the two tangents]

In $\triangle OPA$,

$$\angle OAP + \angle OPA + \angle POA = 180^\circ$$

[Angle sum property of a triangle]

$$\Rightarrow 90^\circ + 40^\circ + \angle POA = 180^\circ$$

$$\Rightarrow 130^\circ + \angle POA = 180^\circ$$

$$\Rightarrow \angle POA = 50^\circ$$

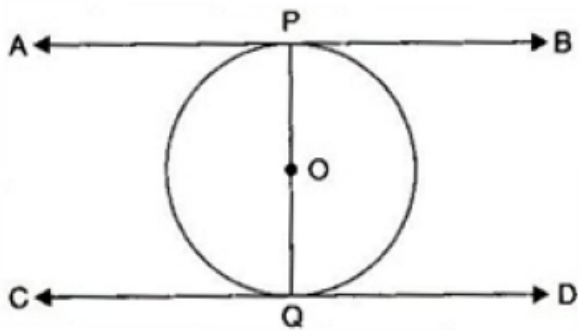
Ex 10.2 Question 4.

Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Answer.

Given: PQ is a diameter of a circle with centre O .

The lines AB and CD are the tangents at P and Q respectively.



To Prove: $AB \parallel CD$

Proof: Since AB is a tangent to the circle at P and OP is the radius through the point of contact.

$$\therefore \angle OPA = 90^\circ$$

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\therefore$ CD is a tangent to the circle at Q and OQ is the radius through the point of contact.

$$\therefore \angle OQD = 90^\circ$$

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

From eq. (i) and (ii), $\angle OPA = \angle OQD$

But these form a pair of equal alternate angles also,

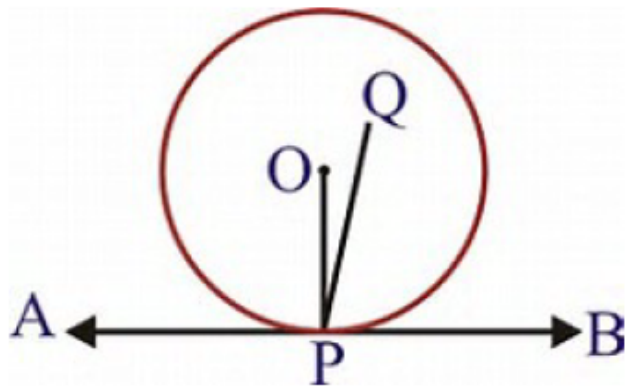
$$\therefore AB \parallel CD$$

Ex 10.2 Question 5.

Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Answer.

Let AB be the tangent drawn at the point P on the circle with O.



If possible, let PQ be perpendicular to AB, not passing through O. Join OP.

Since tangent at a point to a circle is perpendicular to the radius through the point.

$$\text{Therefore, } AB \perp OP \Rightarrow \angle OPB = 90^\circ$$

$$\text{Also, } \angle QPB = 90^\circ$$

[By construction]

Therefore, $\angle QPB = \angle OPB$, which is not possible as a part cannot be equal to whole.

Thus, it contradicts our supposition.

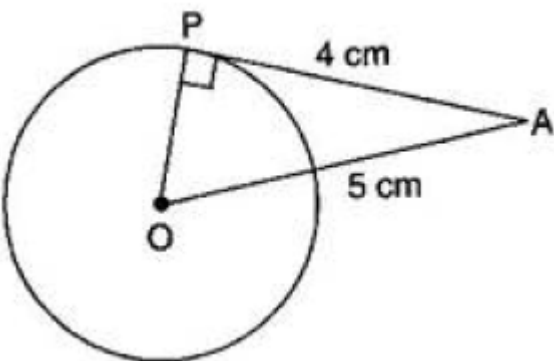
Hence, the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Ex 10.2 Question 6.

The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Answer.

We know that the tangent at any point of a circle is $\perp$ to the radius through the point of contact.



$$\therefore \angle OPA = 90^\circ$$

$$\therefore OA^2 = OP^2 + AP^2$$

[By Pythagoras theorem]

$$\Rightarrow (5)^2 = (OP)^2 + (4)^2$$

$$\Rightarrow 25 = (OP)^2 + 16$$

$$\Rightarrow OP^2 = 9$$

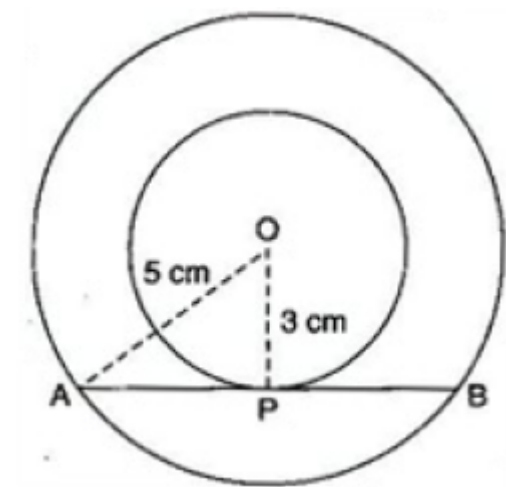
$$\Rightarrow OP = 3 \text{ cm}$$

Ex 10.2 Question 7.

Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer.

Let O be the common centre of the two concentric circles.



Let AB be a chord of the larger circle which touches the smaller circle at P.

Join OP and OA.

Then, $\angle OPA = 90^\circ$

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$$\therefore OA^2 = OP^2 + AP^2$$

[By Pythagoras theorem]

$$\Rightarrow (5)^2 = (3)^2 + AP^2$$

$$\Rightarrow 25 = 9 + AP^2$$

$$\Rightarrow AP^2 = 16$$

$$\Rightarrow AP = 4 \text{ cm}$$

Since the perpendicular from the centre of a circle to a chord bisects the chord, therefore

$$AP = BP = 4 \text{ cm}$$

$$\Rightarrow AB = AP + BP$$

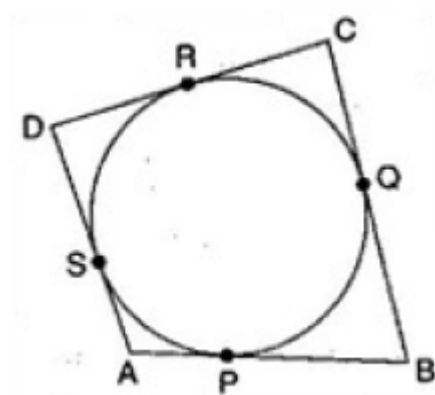
$$= AP + AP = 2AP$$

$$= 2 \times 4 = 8 \text{ cm}$$

Ex 10.2 Question 8.

A quadrilateral ABCD is drawn to circumscribe a circle (see figure). Prove that:

$$AB + CD = AD + BC$$



Answer.

We know that the tangents from an external point to a circle are equal.

$$\therefore AP = AS$$

$$BP = BQ$$

$$CR = CQ$$

$$DR = DS$$

On adding eq. (i), (ii), (iii) and (iv), we get

$$(AP + BP) + (CR + DR)$$

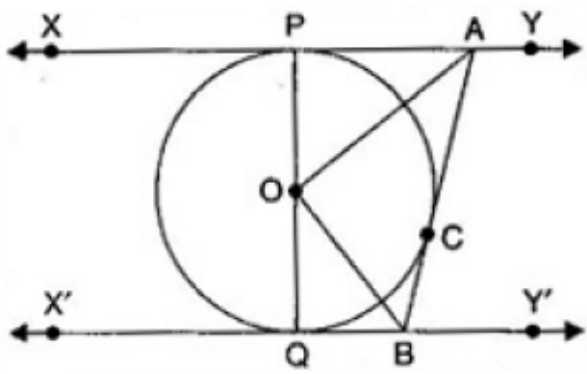
$$= (AS + BQ) + (CQ + DS)$$

$$\Rightarrow AB + CD = (AS + DS) + (BQ + CQ)$$

$$\Rightarrow AB + CD = AD + BC$$

Ex 10.2 Question 9.

In figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle AOB = 90^\circ$.



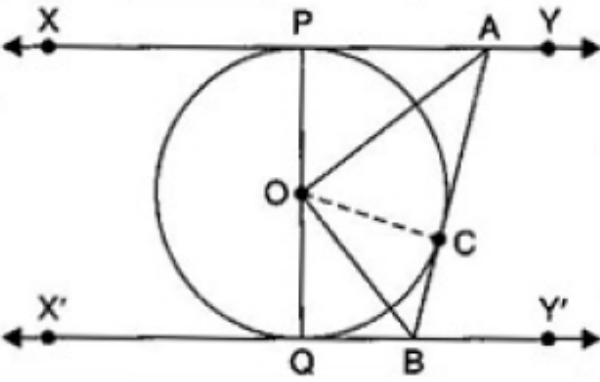
Answer.

Given: In figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B.

To Prove: $\angle AOB = 90^\circ$

Construction: Join OC

Proof: $\angle OPA = 90^\circ$ (i)



$$\angle OCA = 90^\circ$$

[Tangent at any point of a circle is $\perp$ to the radius through the point of contact]

In right angled triangles OPA and OCA,

$$\angle OPA = \angle OCA = 90^\circ$$

$$OA = OA \text{ [Common]}$$

$$AP = AC \text{ [Tangents from an external}$$

point to a circle are equal]

$$\therefore \triangle OPA \cong \triangle OCA$$

[RHS congruence criterion]

$$\therefore \angle OAP = \angle OAC \text{ [By C.P.C.T.]}$$

$$\Rightarrow \angle OAC = \frac{1}{2} \angle PAB$$

$$\text{Similarly, } \angle OBQ = \angle OBC$$

$$\Rightarrow \angle OBC = \frac{1}{2} \angle QBA$$

$\because$ XY $\parallel$ X'Y' and a transversal AB intersects them.

$$\therefore \angle PAB + \angle QBA = 180^\circ$$

[Sum of the consecutive interior angles on the same side of the transversal is 180°]

$$\Rightarrow \frac{1}{2} \angle PAB + \frac{1}{2} \angle QBA$$

$$= \frac{1}{2} \times 180^\circ$$

$$\Rightarrow \angle OAC + \angle OBC = 90^\circ$$

[From eq. (iii) \& (iv)]

In $\triangle AOB$,

$$\angle OAC + \angle OBC + \angle AOB = 180^\circ$$

[Angel sum property of a triangle]

$$\Rightarrow 90^\circ + \angle AOB = 180^\circ \text{ [From eq. (v)]}$$

$$\Rightarrow \angle AOB = 90^\circ$$

Hence proved.

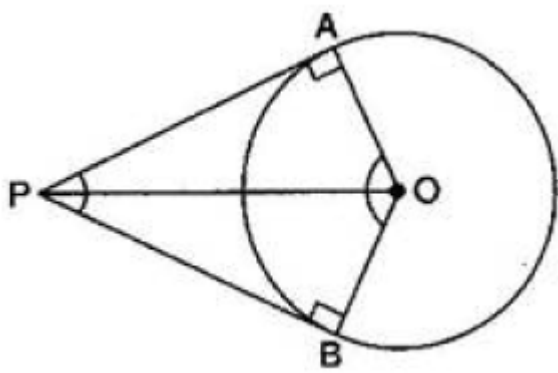
Ex 10.2 Question 10.

Prove that the angel between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Answer.

$$\angle OAP = 90^\circ$$

$$\angle OBP = 90^\circ. \quad \text{(ii)}$$



[Tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\therefore$ OAPB is quadrilateral.

$$\therefore \angle APB + \angle AOB + \angle OAP + \angle OBP = 360^\circ$$

[Angle sum property of a quadrilateral]

$$\Rightarrow \angle APB + \angle AOB + 90^\circ + 90^\circ = 360^\circ$$

[From eq. (i) \& (ii)]

$$\Rightarrow \angle APB + \angle AOB = 180^\circ$$

$\therefore$ $\angle APB$ and $\angle AOB$ are supplementary.

Ex 10.2 Question 11.

Prove that the parallelogram circumscribing a circle is a rhombus.

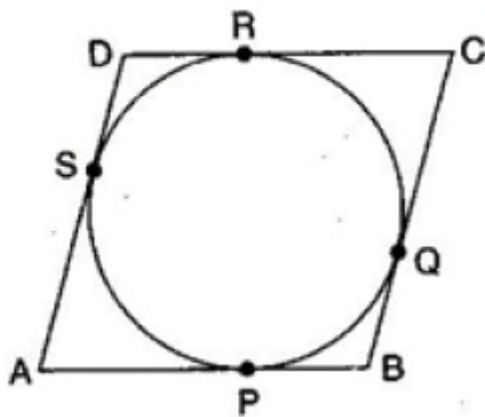
Answer.

Given: ABCD is a parallelogram circumscribing a circle.

To Prove: ABCD is a rhombus.

Proof: Since, the tangents from an external point to a circle are equal.

$$\therefore AP = AS$$



$$BP = BQ$$

$$CR = CQ$$

$$DR = DS$$

On adding eq. (i), (ii), (iii) and (iv), we get

$$(AP + BP) + (CR + DR)$$

$$= (AS + BQ) + (CQ + DS)$$

$$\Rightarrow AB + CD = (AS + DS) + (BQ + CQ)$$

$$\Rightarrow AB + CD = AD + BC$$

$$\Rightarrow AB + AB = AD + AD$$

[Opposite sides of \gm are equal]

$$\Rightarrow 2AB = 2AD$$

$$\Rightarrow AB = AD$$

But $AB = CD$ and $AD = BC$

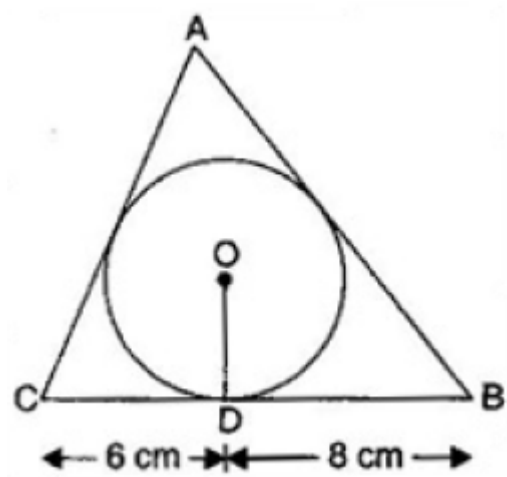
[Opposite sides of \gm]

$$\therefore AB = BC = CD = AD$$

$\therefore$ Parallelogram ABCD is a rhombus.

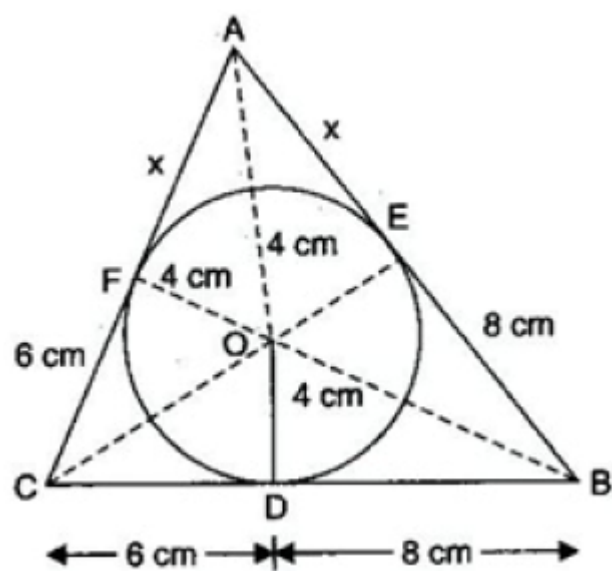
Ex 10.2 Question 12.

A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see figure). Find the sides AB and AC.



Answer.

Join OE and OF. Also join OA, OB and OC.



Since $BD = 8$ cm

$\therefore BE = 8$ cm

[Tangents from an external point to a circle are equal]

Since $CD = 6$ cm

$\therefore CF = 6$ cm

[Tangents from an external point to a circle are equal]

Let $AE = AF = x$

Since $OD = OE = OF = 4$ cm

[Radii of a circle are equal]

$$\therefore \text{Semi-perimeter of } \triangle ABC = \frac{(x+6) + (x+8) + (6+8)}{2} = (x+14)\text{cm}$$

$$\begin{aligned} \therefore \text{Area of } \triangle ABC &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{(x+14)(x+14-14)(x+14-x-8)(x+14-x-6)} \\ &= \sqrt{(x+14)(x)(6)(8)}\text{cm}^2 \end{aligned}$$

Now, Area of $\triangle ABC = \text{Area of } \triangle OBC + \text{Area of } \triangle OCA + \text{Area of } \triangle OAB$

$$\begin{aligned} &\Rightarrow \sqrt{(x+14)(x)(6)(8)} \\ &= \frac{(6+8)4}{2} + \frac{(x+6)4}{2} + \frac{(x+8)4}{2} \\ &\Rightarrow \sqrt{(x+14)(x)(6)(8)} \\ &= 28 + 2x + 12 + 2x + 16 \\ &\Rightarrow \sqrt{(x+14)(x)(6)(8)} = 4x + 56 \\ &\Rightarrow \sqrt{(x+14)(x)(6)(8)} = 4(x+14) \end{aligned}$$

Squaring both sides,

$$\begin{aligned} (x+14)(x)(6)(8) &= 16(x+14)^2 \\ \Rightarrow 3x &= x+14 \\ \Rightarrow 2x &= 14 \\ \Rightarrow x &= 7 \end{aligned}$$

$$\therefore AB = x + 8 = 7 + 8 = 15 \text{ cm}$$

$$\text{And } AC = x + 6 = 7 + 6 = 13 \text{ cm}$$

Ex 10.2 Question 13.

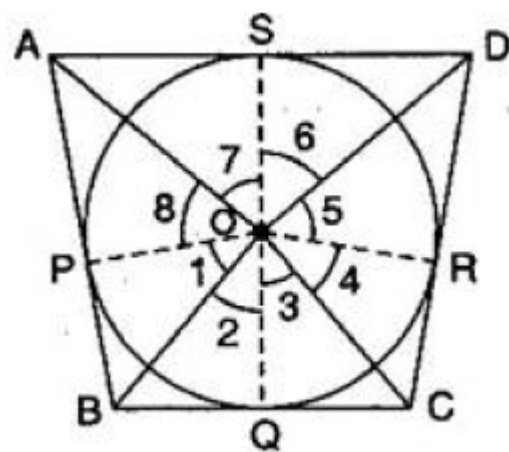
Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Answer.

Given: ABCD is a quadrilateral circumscribing a circle whose centre is O.

To prove: (i) $\angle AOB + \angle COD = 180^\circ$ (ii) $\angle BOC + \angle AOD = 180^\circ$

Construction: Join OP, OQ, OR and OS.



Proof: Since tangents from an external point to a circle are equal.

$\therefore AP = AS,$

$BP = BQ \dots$

$CQ = CR$

$DR = DS$

In $\triangle OBP$ and $\triangle OBQ$,

$OP = OQ$ [Radii of the same circle]

$OB = OB$ [Common]

$BP = BQ$ [From eq. (i)]

$\therefore \triangle OPB \cong \triangle OBQ$ [By SSS congruence criterion]

$\therefore \angle 1 = \angle 2$ [By c.P.C.T.]

Similarly, $\angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8$

Since, the sum of all the angles round a point is equal to 360° .

$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$

$\Rightarrow \angle 1 + \angle 1 + \angle 4 + \angle 4 + \angle 5 + \angle 5 + \angle 8 + \angle 8 = 360^\circ$

$\Rightarrow 2(\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360^\circ$

$\Rightarrow \angle 1 + \angle 4 + \angle 5 + \angle 8 = 180^\circ$

$\Rightarrow (\angle 1 + \angle 5) + (\angle 4 + \angle 8) = 180^\circ$

$\Rightarrow \angle AOB + \angle COD = 180^\circ$

Similarly, we can prove that

$\angle BOC + \angle AOD = 180^\circ$